

Normalerichteter

~~$t = (\dots ((\lambda x, x) y) \dots)$~~

$t = (\dots ((\lambda x, \dots) y) \dots)$

Linke. Äußerste  
↓

Linke. Äußerste  
↓

äußere  
↓

$(\lambda y. (\lambda x. \dots) y) \dots$   
NF

$(\lambda y. (\lambda x. \dots) y) z$

~~$(\lambda x. x) (\lambda x. x)$~~

$(\lambda z. ((\lambda x. z) (\lambda y. z))) ((\lambda x. \dots) y)$

25

$((\lambda a.a) (\lambda b.b)) ((\lambda c.c) ((\lambda d.d) (\lambda e.e) (f.f))) (\lambda g.g) ((\lambda h.h) (\lambda i.i))$

3

$(\lambda f. \lambda x. f) (\lambda y. (\lambda x.x x) (\lambda x.x x)) ((\lambda x.x) (\lambda x.x)) (\lambda f. \lambda f. f)$

NF/CBV

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CBV

$\lambda y. (\lambda z. (\lambda x.x) (\lambda x.x) z) \gamma$   
~~~~~  
~~~~~

NF

4

$$\text{pair} = \lambda a. \lambda b. \lambda f. f a b$$

Sup: pair c<sub>0</sub> c<sub>time</sub> ⇒ λf. c<sub>0</sub> c<sub>time</sub>

$$\text{fst} = \lambda p. p (\lambda a. \lambda b. a)$$

1

$$\text{snd} = \lambda p. p (\lambda a. \lambda b. b)$$

2

$$\text{fst} (\text{pair } a \ b) \Rightarrow (\lambda p. p (\lambda a. \lambda b. a)) \ (\underline{(\lambda a. \lambda b. \lambda f. f a b) a b})$$

$$\Rightarrow (\underline{\lambda a. \lambda b. \lambda f. f a b}) \ \underline{a} \ b \ (\lambda a. \lambda b. a)$$

$$\Rightarrow (\underline{\lambda f. f a b}) \ \underline{b} \ (\lambda a. \lambda b. a)$$

$$\Rightarrow (\underline{\lambda f. f a b}) \ (\underline{\lambda a. \lambda b. a})$$

$$\Rightarrow (\underline{\lambda a. \lambda b. a}) \ \underline{a} \ b$$

$$\Rightarrow (\underline{\lambda b. a}) \ b$$

$$\Rightarrow a$$

$$\boxed{3} \quad \text{next } (n, m) \rightsquigarrow (n, n+1)$$

$$\text{next} = \lambda p. \text{pair } (\text{snd } p) \quad (\text{succ } (\text{snd } p))$$

VL:

$$\text{succ} = \lambda n. \lambda s. \lambda z. s (n s z)$$

$$\boxed{4} \quad \text{next } (\text{pair } n \ m) \implies \text{pair } (\text{snd } (\text{pair } n \ m)) \quad (\text{succ } (\text{snd } (\text{pair } n \ m)))$$

$$\implies^* \text{pair } m \quad (\text{succ } m)$$

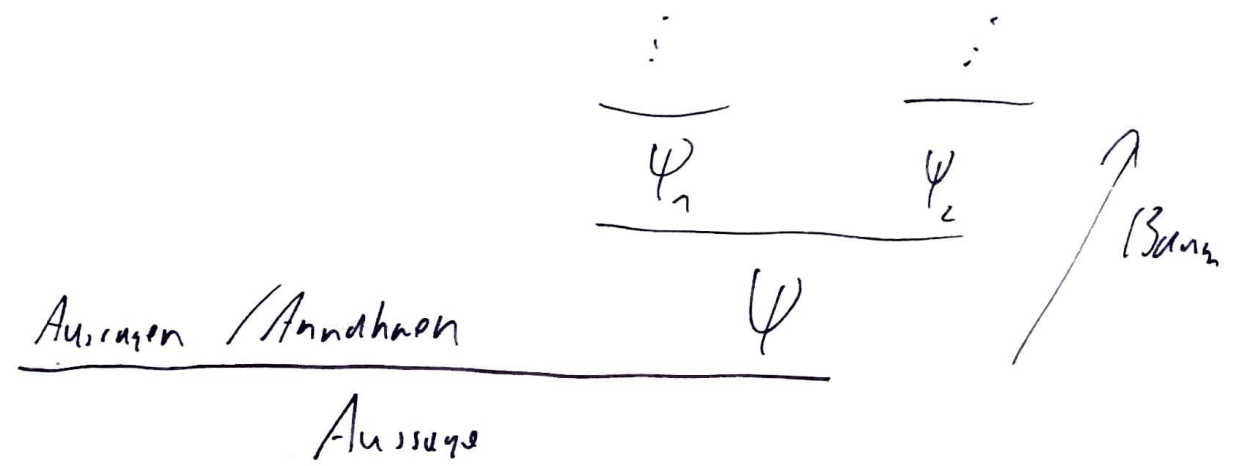
$$\boxed{5} \quad \text{pred} = \lambda n \ \text{fst} \ (n \ \text{next} \ (\text{pair } c_0 \ c_0))$$

$$\text{pred } c_n \rightsquigarrow \text{fst } (\text{next}^n \ (\text{pair } c_0 \ c_0))$$

$$\boxed{7} \quad \text{sub} = \lambda m. \lambda n \ n \ \text{pred } m$$

$$\text{sub } c_m \ c_n \rightsquigarrow \text{pred}^n \ c_m$$

Rechtssysteme



Bsp.:

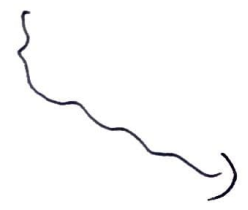
$$MP \frac{\Gamma \vdash \psi \rightarrow \psi \quad \Gamma \vdash \psi}{\Gamma \vdash \psi}$$

Knollen gewächse wachsen  
 unter der Erde

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Knollen sind  
 Knollen gewächse

Knollen wachsen unter der Erde



???

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Funktion  $f$  hat Typ  $\alpha$

6/17

$$\text{pair} : (\alpha \rightarrow \beta \rightarrow (\alpha - \beta \rightarrow \gamma)) \rightarrow \gamma$$

Ubergang  $\Gamma = \{x: \text{int} \rightarrow \text{int} \rightarrow \text{int}\}$

Regeln: ABS      VAR  
          APP      CONST

VAR  
⑥

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$$\Gamma, x: \text{int}, f: (\text{int} \rightarrow \text{bool}) \vdash f : (\text{int} \rightarrow \text{bool}) \quad | \quad \Gamma, x: \text{int}, f: (\text{int} \rightarrow \text{bool}) \vdash (+ \times 5) : \text{int}$$

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APP  
③ ⑤

$$\Gamma, x: \text{int}, f: (\text{int} \rightarrow \text{bool}) \vdash f (+ \times 5) : \text{bool}$$

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ABS  
④ ⑥

$$\Gamma, x: \text{int} \vdash (\lambda p. f (+ \times 5)) : (\text{int} \rightarrow \text{bool}) \rightarrow \text{bool}$$

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ABS  
① ③

$$\Gamma \vdash (\lambda x. \lambda p. f (+ \times 5)) : \text{int} \rightarrow (\text{int} \rightarrow \text{bool}) \rightarrow \text{bool}$$

(11) Var

$$\frac{\Gamma'(x) = \text{int} \quad \Gamma'(+) = \text{int} \rightarrow \text{int} \rightarrow \text{int}}{\Gamma' \vdash + : \text{int} \rightarrow \text{int} \rightarrow \text{int}} \quad \Gamma' \vdash x : \text{int}$$

~~VAR~~ VAR (12)

(13) APP

$$\frac{\Gamma' \vdash (+ x) : \text{int} \rightarrow \text{int} \quad \Gamma' \vdash 5 : \text{int}}{\Gamma' \vdash (+ x 5) : \text{int}}$$

~~VAR~~ CONST VAR (14) (15)

(14) APP

$$\Gamma', x : \text{int}, f : (\text{int} \rightarrow \text{bool}) \vdash (+ x 5) :$$

$\Gamma'$

CONST

$\Gamma \vdash x: \alpha \vdash z: \text{int}$

AB1

$\Gamma \vdash \lambda x. z: (\alpha \rightarrow \text{int})$

LET

$\Gamma \vdash \text{let } f = \lambda x. z \text{ in } f(f \text{ true}): \text{int}$

$\forall \alpha. \Gamma \vdash f: \alpha \rightarrow \text{int}$   
 $\forall \alpha. \alpha \rightarrow \text{int} \geq \text{int} \rightarrow \text{int}$   
VAR

$\Gamma \vdash f: \text{int} \rightarrow \text{int}$

$\forall \alpha. \Gamma \vdash f: \alpha \rightarrow \text{int}$   
 $\forall \alpha. \alpha \rightarrow \text{int} \geq \text{int} \rightarrow \text{int}$

Var  $\Gamma \vdash f: \text{bool} \rightarrow \text{int}$

$\text{true}: \text{bool}$  CONST

$f \text{ true}: \text{int}$

APP

$\Gamma' = \{f: \alpha \rightarrow \text{int}, \Gamma\} \cup \Gamma$

$f: \forall \alpha \alpha \rightarrow \text{int} \vdash f(f \text{ true}): \text{int}$

LET